The Electromagnetic Rocket Gun Impact Fusion Driver

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An impact fusion driver is discussed which has the potential to accelerate a rocket-like small metallic projectile by a travelling magnetic wave to a few 100 km/sec over a reasonable distance. The propellant removes the heat produced in the projectile by resistive losses of the induced electric currents, and is thereby ejected from the projectile at a modest temperature. After having left the projectile, the propellant is shock-heated by the strong magnetic field of the travelling magnetic wave to high temperatures forming a high velocity jet. The total thrust on the projectile is the sum of the magnetic and recoil forces. In comparison to a rocket, the efficiency is much larger, with the momentum transferred to the gun barrel rather than to a tenuous jet.

To reach impact fusion velocities the accelerator would be a few 100 m long.

1. Introduction

The first drivers proposed for inertial fusion (proposed even prior to laser- or electron- or ion-beam drivers) have been macroparticle accelerators, capable of accelerating small projectiles to velocities of a few 100 km/sec [1]. The principle of this inertial fusion concept, also known under the name of impact fusion, is explained in Figure 1. A projectile P coming from the left at high velocity strikes a fusion fuel embedded in a conical cavity of an anvil. First, upon impact, the fusion fuel is shock-heated to high temperatures, followed by an almost adiabatic compression and heating until ignition with subsequent thermonuclear burn. The conical implosion configuration ensures fast compression, minimizing the impact velocity needed to overcome the bremsstrahlung-losses.

A theoretical analysis of this configuration done by the author may years ago, showed that the minimum velocity required would be ~ 200 km/sec [2]. This result, obtained by a similarity solution, has been later confirmed by other researchers using numerical methods [1].

The advantage of impact fusion over other inertial confinement fusion concepts is that it permits a much smaller target compression (less than 100 fold) compared to a ~ 1000 fold compression needed with beam drivers. The reason for the substantial reduction in the required compression is that in impact fusion the target is strongly tamped by the high velocity projectile, whereas in beam induced inertial fusion the target must be compressed ablatively. Another important advantage of impact fusion is its ability to make for a large stand-off distance between the reactor wall and the microexplosion.

The only known means by which the high projectile velocities required for impact fusion can be reached are magnetic accelerators.

One version of these magnetic macroparticle accelerators is the magnetic dipole accelerator. Its working principle is shown in Figure 2. It consists of many magnetic field coils, also called driver coils, each one activated by a current pulse from a capacitor. The projectile to be accelerated inside these coils is a small magnetic dipole. It can be a ferromagnetic projectile, a small superconducting solenoid, or even a good conductor. In case the projectile is a ferromagnet or superconductor, it has a permanent dipole moment impressed on it. In case

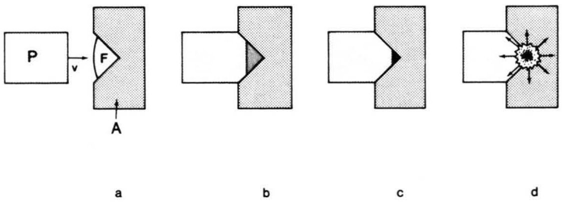

Fig. 1. Impact Fusion Concept and Sequence of Events: Projectile P strikes anvil A holding thermonuclear fuel F in conical cavity, the configurations **a–d**, show, Fig. 1 **a**, moment before impact, **b** shock heating, **c** isentropic compression up to ignition, and **d** thermonuclear burn.

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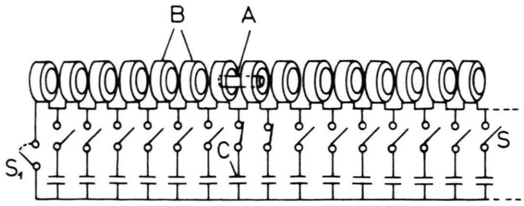

Fig. 2. Magnetic Dipole-Type Wave Accelerator: The Projectile A, which can be a small ferromagnetic rod or superconducting solenoid is magnetically accelerated through external magnetic field coils B. C are capacitors and $S_1, \ldots, S$ are switches to be closed as the projectile moves down the accelerator tube.

it is an ordinary conductor, it acquires a dipole moment by electric currents induced in it through the rapidly rising magnetic field of the driver coils. The acceleration of the projectile is achieved through synchronizing the closing of the switches activating the driver coils. The switches must be closed in the moment the projectile position coincides with the position of the driver coils.

The one distinctive property the magnetic dipole accelerator has over other magnetic acceleration schemes, is that the projectile does not have to be in physical contact with the structure of the accelerator. Because only then is it possible to reach, in principle at least, arbitrarily high velocities. In those other magnetic acceleration devices, for example the railgun, where the projectile is in physical contact with the wall, friction losses lead to a boundary layer separating the projectile from the wall, which at large velocities radiates at a very high rate. The temperature of the boundary layer rises with the 2nd power of the projectile velocity. Black body radiation losses, which are proportional to the 4th power of the temperature, would therefore rise with the 8th power of the velocity. This means that the losses would become intolerable above a certain velocity. Estimates show (see Appendix II), why because of these losses only dipole-type magnetic accelerators have a chance to reach the high velocities of a few 100 km/sec as they are required for impact fusion.

The idea to launch a ferromagnetic projectile by a travelling magnetic wave can be already found in the works of Oberth [3]. An electromagnetic launcher, using a superconducting projectile was proposed by Maisonnier [4] and the author many years ago [5, 6]. More recently, Garwin, Muller and Richter [7] have proposed to use high saturation field strength ferromagnetic projectiles, made up of such exotic substances as gadolinium or holmium.

One common problem of all magnetic dipole accelerators has to do with the switching of the driver coils. The length of the driver coils must be about equal to the length of the projectile to be accelerated. For impact fusion the projectile should be not larger than ~ 1 cm. At a projectile velocity in excess of 10^7 cm/sec, the switching would therefore have to be done very fast, in less than $\sim 10^{-7}$ sec.

The magnetic force accelerating the projectile in the z-direction is given by (in electrostatic-cgs-units)

$$F = M \, \mathrm{d}H/\mathrm{d}z \,, \tag{1.1}$$

where M is the magnetic moment of the projectile. If the magnetic field produced by the dipole moment of the projectile is H_p and the projectile volume V, one has

$$M = H_\mathrm{p} \, V/12\,\pi \,. \tag{1.2}$$

Then, if the density of the projectile is ϱ, its acceleration is $a = F/\varrho \, V$, hence

$$a = H_\mathrm{p} (\mathrm{d}H/\mathrm{d}z) \cdot (1/12\,\pi\,\varrho) \,. \tag{1.3}$$

The maximum possible magnetic field gradient is reached if the magnetic field rises from $-H$ to $+H$ over the length of the projectile, hence

$$(\mathrm{d}H/\mathrm{d}z)_\mathrm{max} \simeq 2\,H/l \,. \tag{1.4}$$

One therefore has

$$a = H_\mathrm{p} H/6\,\pi\,\varrho\,l \,. \tag{1.5}$$

In case $H_\mathrm{p} \simeq H$, which, for example, is realized with a good conductor which is shielding the externally applied magnetic field through surface currents from its interior, one has

$$a = H^2/6\,\pi\,\varrho\,l \,. \tag{1.6}$$

Equation (1.6) has a simple interpretation in terms of the magnetic pressure which is given by $H^2/8\,\pi$. Because, if a magnetic pressure of magnitude $H^2/8\,\pi$ would act on the rearside of the projectile, this pressure would excert on it a force equal to $F = (H^2/8\,\pi)\,\pi\,r^2$. The magnetic pressure would therefore lead to an acceleration given by

$$a = F/\pi\,r^2\,\varrho\,l = H^2/8\,\pi\,\varrho\,l \,, \tag{1.7}$$

and which is about the same as (1.6).

Unfortunately, due to the tensile strength of the driver coils and projectile, the magnetic field cannot be made arbitrarily large. If the tensile strength of the coils (or projectile) is σ_s, equating the magnetic shear stress $H^2/4\pi$ with σ_s leads to

$$H = H_{max} = \sqrt{4\pi\sigma_s}\,. \quad (1.8)$$

For $\sigma_s \simeq 10^{10}$ dyn/cm^2, which is typical for steel but also for other high tensile strength materials, one finds $H_{max} \simeq 3 \times 10^5$ G.

In addition, the maximum possible acceleration is not determined by the tensile strength alone, but also by the length of the projectile. The acceleration a produces inside the projectile a pressure gradient, with the maximum pressure given by

$$p_{max} = \varrho a l\,, \quad (1.9)$$

where ϱ is the density of the projectile material and l its length. Equating p_{max} with σ_s leads to

$$a_{max} = \sigma_s / l\varrho = (H^2_{max}/4\pi)/\varrho l\,. \quad (1.10)$$

Equation (1.10) shows that the acceleration increases with decreasing l. However, it is not possible to make l arbitrarily small, because this would require to make the magnetic field gradient arbitrarily large, which is assumed to be of the order H/l. The acceleration also increases with decreasing ϱ, but there is, of course, a practical lower limit on ϱ. With (1.10) the minimum length L of a magnetic accelerator to reach a projectile velocity v is given by

$$L = v^2/2a_{max} = l\varrho v^2/2\sigma_s = 2\pi l\varrho(v/H_{max})^2\,. \quad (1.11)$$

Assume, for example, that $\varrho \simeq 3$ g/cm^3 (valid for some light metal), $l \simeq 1$ cm, and $v = 2 \times 10^7$ cm/sec. One there finds $L \simeq 600$ meters.

One drawback, shared by both superconducting and ferromagnetic projectiles is, that they cannot be accelerated with the maximum magnetic force permitted by the tensile strength. The maximum magnetic field in either case is about 5–10 times smaller than what the tensile strength would permit, resulting in a ~ 25 to 100 times smaller acceleration, and a 25 to 100 times larger length of the accelerator. This would mean an accelerator from 15–60 km long. In reality, the accelerator would be even longer, because both superconductors and high-field ferromagnets have a rather high density. An accelerator 100 km long would therefore be a more realistic estimate.

Only ordinary conductors permit magnetic body forces which reach the upper limit set by their tensile strength, and they also can be made of light metals, for example aluminum. Now however, a new problem arises, because there the projectile acquires its large magnetic moment through induced electric currents. These large electric currents heat the projectile by resistive losses, with the result that it vaporizes before reaching a high velocity. This problem is fatal for small cm-size macroparticles, and an estimate of the maximum attainable velocity under these circumstances is given in Appendix III. It falls short of what is needed for impact fusion. Would it not be for the vaporization problem, magnetically accelerated light-weight ordinary conductors would make the shortest magnetic macroparticle accelerators.

8. Electromagnetic Rocket Gun

To overcome the different problems for magnetic dipole accelerators outlined in the introduction, we propose a novel accelerator concept called electromagnetic rocket gun. To reach the maximum possible magnetic body force permitted by the tensile strength, it uses ordinary conductors. But in addition, the projectile to be accelerated by the magnetic forces is now also a rocket. As in a rocket, where the propellant cools the rocket and thereby prevents it from burning up, the same happens here, except that the heat is produced by resistive energy losses, not by a chemical reaction. In addition to serve as a heat sink, the propellant of the electromagnetic rocket gun itself is also resistively heated by the magnetic field pushing against the projectile. This results in a large exhaust velocity of the propellant adding to the thrust of the magnetic body force accelerating the projectile. But unlike a rocket, the propellant receives its energy externally. For a given magnetic field strength the resistive heating rate goes up in proportion to the projectile velocity and by which the exhaust velocity also goes up with the projectile velocity. Also unlike a rocket, the recoil momentum is transmitted through the magnetic force field to the gun barrel rather than the exhaust. Because of both of these effects, first the increase in the exhaust velocity, with the projectile velocity, and second the momentum transfer to the massive gun barrel, the acceleration is much more efficient than for a rocket where the exhaust velocity is constant with the recoil momentum going into a tenuous jet.

The propulsion of the projectile is caused by the same kind of mechanism as in hybrid arc-magneto-gasdynamic rocket propulsion [8]. The thermal arc-heating alone would produce an exhaust velocity w_g given by

$$w_g^2 = 2\,h\,, \tag{2.1}$$

where h is the enthalpy of the arc-heated gas. For an ideal gas of specific heat at constant pressure c_p, heated by the arc to the temperature T, one has $h = c_p T$, and hence

$$w_g^2 = 2\,c_p\,T\,. \tag{2.2}$$

However, it was shown by Maecker [9], that for an arc-heated plasma expanding in a magnetic field there is in addition to the pure gasdynamic force a magnetohydrodynamic body force. It alone would lead to a plasma velocity w_H given by

$$w_H^2 \sim I\,j/\varrho\,c^2\,, \tag{2.3}$$

where I and j are the total electric current and current density in the arc plasma, both measured in electrostatic cgs units. With the help of Maxwell's equation $(4\,\pi/c)\,\boldsymbol{j} = \operatorname{curl}\boldsymbol{H}$, where $\boldsymbol{H}$ is the magnetic field, (2.3) takes the form

$$w_H^2 \sim H^2/4\,\pi\,\varrho\,. \tag{2.4}$$

The arc occurs because the magnetic field diffuses into the plasma and which results in resistive heating. If the magnetic field has completely penetrated the plasma, the enthalpy per volume, $\varrho\,c_p\,T$, of the plasma is equal to the magnetic energy density $H^2/8\,\pi$, hence

$$\varrho\,c_p\,T = H^2/8\,\pi\,. \tag{2.5}$$

From (2.5) follows that

$$w_H^2 \sim w_g^2\,. \tag{2.6}$$

The total exhaust velocity w is therefore given by

$$w^2 = w_g^2 + w_H^2 \sim 2\,w_g^2 \tag{2.7}$$

and which shows that the thrust is twice as large if only one of the forces would act alone.

Before entering the region where the arc burns, the gas is pre-heated within the projectile by the transfer of the heat produced by the resistive losses inside the projectile. If not cooled by the propellant, the projectile would burn up as a result of this resistive heating. We therefore may assume a projectile temperature of $\sim 10^3$ K, just below the melting point of the projectile material. If, for example, the propellant is hydrogen, it would at such a temperature be expelled from the projectile with a velocity of a few km/sec, for example $v_0 = 3 \times 10^5$ cm/sec. In entering the magnetic field, an electric field E is induced inside the gas moving with the velocity v_0 into the magnetic field, and which is given by (measured in electrostatic cgs units)

$$E = (v_0/c)\,H\,, \tag{2.8}$$

where H is the magnetic field strength measured in Gauss. If, for example, $H \sim 10^5$ G, it would follow that $E \sim 1$ esu = 300 Volt/cm. This voltage is more than enough to start by electric breakdown an arc in the gas entering the magnetic field.

One further important advantage this new concept has in comparison with other types of magnetic travelling wave accelerators, is that because the plasma jet can be much longer than the projectile, the length of the travelling magnetic wave can be made much longer too. The time needed to turn on the magnetic field can therefore become much longer, eliminating the problem associated with the short times to turn on the field. If, for example, the projectile moves with a velocity of 100 km/sec = 10^7 cm/sec, and if the length of the plasma jet is 30 cm, the switching time would be increased from $\sim 10^{-7}$ sec to 3×10^{-6} sec.

According to (1.10), the larger length l, of the rocket-like projectile, would reduce the maximum possible acceleration but this is offset by the low density of the propellant, reducing the average density of the overall projectile assembly. The small average density is then possible, if the propellant is subjected to pressure forces only, with the encasement balancing the remaining shear stresses.

A way how the electromagnetic rocket gun can be realized is shown in Figure 3. A conducting metallic projectile P, being hollowed out except for its payload section PL, is placed in front of a travelling magnetic wave. The hollowed out part of the projectile is filled with some propellant F, preferably solid hydrogen. Hydrogen is the optimal propellant because its radiation losses are low at the temperatures reached. The projectile is placed in front of a travelling magnetic wave, generated by the field coils MC. The rising magnetic field at the front of

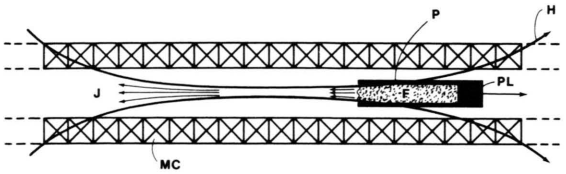

Fig. 3. Electromagnetic rocket gun principle. P part of the projectile holding the propellant F which vaporizes and together with the propellant becomes part of the jet J. PL projectile payload, MC magnetic field coils, H magnetic lines of force.

the travelling magnetic wave induces surface currents in the back part of the hollowed out projectile. The dissipation of the currents produces heat by which the solid hydrogen propellant is slowly vaporized. The vaporized hydrogen thereafter enters the region behind the projectile where the magnetic field rapidly rises. In entering this region, the hydrogen gas is first shock-heated to high temperatures and becomes ionized. The rising magnetic field further compresses and heats the hydrogen plasma, until it has reached the position of the travelling magnetic wave where the field strength is at its maximum value, and where the magnetic field has the shape of a magnetic mirror. In passing through the magnetic mirror the hot hydrogen plasma is ejected as a jet J with high speed. The projectile is accelerated because of both the magnetic pressure force and the recoil force of the jet. During its acceleration the projectile loses mass and it therefore behaves in part like a rocket. But because the projectile is also accelerated by the magnetic pressure force, it also acts in part like a projectile fired by a magnetic gun. In addition to the propellant, the metallic encasement holding the propellant, too is slowly ablated and ejected, thereby also contributing to the recoil-produced thrust. Both, the high conductivity of the encasement and of the plasma, ensures that the magnetic pressure by the travelling wave can exert its full force on the projectile.

To prevent the projectile from touching the wall, and which would result in disastrous black body radiation losses, it must be made from some ferromagnetic material. By magnetic feedback control it can then be steered to stay away from the wall.

The continuous evaporation of the propellant prevents the projectile from being melted, and by a proper choice of the thickness and conductivity of the metal encasing the propellant one can arrange that the Joule heat produced in the encasement by the travelling magnetic wave gives the right evaporation rate of the propellant.

The concept differs in at leat three significant points from a rocket: First, the energy is here supplied externally. Second, the travelling wave produces a magnetic pressure force, in addition to the recoil-force produced by the hot plasma jet. Third, unlike in a real rocket, the nozzle is here externally established by the magnetic field, radially confining the plasma jet.

The concept has, of course, also similarities with electromagnetic guns, but it significantly differs from those guns because the projectile also behaves like a rocket. It is for these various reasons, making it quite unique, that we call this macron-accelerator an electromagnetic rocket gun.

3. The Acceleration Physics

The magnetic body force density $\boldsymbol{f}$ acting on the projectile is given (in electrostatic units) by

$$\boldsymbol{f} = (1/c)\, \boldsymbol{j} \times \boldsymbol{H}\,, \tag{3.1}$$

where $\boldsymbol{j}$ is the current density vector, $\boldsymbol{H}$ the magnetic field and c the velocity of light. From Maxwell's equation

$$(4\,\pi/c)\,\boldsymbol{j} = \operatorname{curl} \boldsymbol{H} \tag{3.2}$$

we obtain

$$\boldsymbol{f} = -\frac{1}{4\,\pi}\, \boldsymbol{H} \times \operatorname{curl} \boldsymbol{H}\,. \tag{3.3}$$

With the vector identity

$$\boldsymbol{H} \times \operatorname{curl} \boldsymbol{H} = (1/2)\, \nabla H^2 - (\boldsymbol{H} \cdot \nabla)\, \boldsymbol{H} \tag{3.4}$$

and because in our configuration $\nabla |H|$ is perpendicular to the direction of $\boldsymbol{H}$, making $(\boldsymbol{H} \cdot \nabla)\, \boldsymbol{H} = 0$, we have

$$\boldsymbol{f} = -\nabla\, (\boldsymbol{H}^2/8\,\pi)\,. \tag{3.5}$$

In (3.5) $H^2/8\,\pi$ is the usual expression of the magnetic pressure in the Maxwell stress tensor.

The maximum magnetic force F_{H} acting on the rear side of the projectile having the cross section A is given by

$$F_{\mathrm{H}} = A H^2/8\,\pi\,. \tag{3.6}$$

After the vaporized and preheated propellant has left the rearside of the projectile, it is, like in a theta pinch, shock-heated in entering the region occupied by the strong magnetic field. Because the magnetic field is very strong the propellant becomes a plasma. If the atomic number density of the plasma is n, and if all the magnetic energy, with an energy density $H^2/8\pi$, is converted into heat, one has (see Appendix I)

$$3\,n\,kT = H^2/8\,\pi\,, \tag{3.7}$$

valid for a singly ionized plasma, for example a fully ionized hydrogen plasma.

After it is heated to the temperature given by (3.7), the plasma becomes a jet. The jet is radially confined, both by the magnetic field and the wall of the gun barrel.

The jet produces a recoil force acting on the projectile which is given by

$$F_{\text{rec}} = -\,v\,\mathrm{d}m/\mathrm{d}t\,, \tag{3.8}$$

where v is the jet velocity relative to the projectile and $\mathrm{d}m/\mathrm{d}t$ the mass ejected per unit time. If ϱ is the density of the jet one has

$$\mathrm{d}m/\mathrm{d}t = -\,A\,\varrho\,v\,, \tag{3.9}$$

and hence

$$F_{\text{rec}} = A\,\varrho\,v^2\,. \tag{3.10}$$

If one neglects the electron mass against the ion mass one has

$$3\,n\,kT = \varrho\,v^2 = H^2/8\,\pi\,, \tag{3.11}$$

and therefore

$$F_{\text{rec}} = AH^2/8\,\pi = F_{\text{H}}\,. \tag{3.12}$$

The total force F acting on the rear side of the projectile is the sum

$$F = F_{\text{H}} + F_{\text{rec}} = 2\,F_{\text{rec}}\,. \tag{3.13}$$

With the mass flow rate given by (3.9) one has

$$F = 2v\,\mathrm{d}m/\mathrm{d}t\,. \tag{3.14}$$

We now make the special assumption that in each moment during the acceleration of the projectile the jet velocity v is equal to the projectile velocity. We show later that this assumption can be satisfied by a properly programmed evaporation rate of the propellant. If the jet velocity is always equal to the projectile velocity, then, as seen from a frame at rest with the magnetic gun, the jet comes to rest and no kinetic energy is lost into the jet. Under this special mode of operation the efficiency of the rocket drive is always maximized.

The equation of motion of the projectile is now given by

$$m\,\mathrm{d}v/\mathrm{d}t = F = -\,2\,v\,\mathrm{d}m/\mathrm{d}t\,, \tag{3.15}$$

resulting in the differential equation

$$m\,\mathrm{d}v = -\,2\,v\,\mathrm{d}m\,. \tag{3.16}$$

By integration of (3.16) we find

$$v/v_0 = (m_0/m)^2\,, \tag{3.17}$$

where m_0 is the initial projectiles mass at the time $t = 0$, and where $v = v_0$ is the initial projectile velocity. Comparing (3.17) with the rocket equation $v = v_0 \ln(m_0/m)$, where v_0 is there the constant exhaust velocity, we see that for the same mass ratio m_0/m, much higher velocities can be reached with the electromagnetic rocket gun than with rockets. If, for example, $v_0 = 1$ km/sec and $m_0/m = 10$, then $v = 100$ km/sec is possible, whereas for a rocket $v \simeq 2.3$ km/sec only. Of course, the electromagnetic rocket gun needs a gun barrel and the affordable length of this gun barrel sets a practical upper limit for the attainable velocity.

If we assume that the total force $F = AH^2/4\,\pi$, is constant, the equation of motion of the projectile can be integrated in closed form. We have

$$\mathrm{d}v/\mathrm{d}t = F/m = (F/m_0)\,(v/v_0)^2 \tag{3.18}$$

from which we obtain the first integral

$$v = v_0\,[1 + (F/2\,m_0\,v_0)\,t]^2 \tag{3.19}$$

with the asymptotic expression for large t:

$$v \rightarrow (1/v_0)\,(F/2\,m_0)^2\,t^2\,. \tag{3.20}$$

If z is the distance the projectile has moved down the magnetic gun barrel, with $z = 0$, at $t = 0$ and $v = v_0$, a second integral is obtained from (3.19), after putting $v = \mathrm{d}z/\mathrm{d}t$:

$$\begin{aligned} z &= v_0 \int_0^t [1 + (F/2\,m_0\,v_0)\,t]^2\,dt \\ &= (2/3)\,(m_0\,v_0^2/F)\,[(1 + (F/2\,m_0\,v_0)\,t)^3 - 1] \end{aligned} \tag{3.21}$$

with the asymptotic expression

$$z \rightarrow (1/12)\,(F^2/m_0^2\,v_0)\,t^3\,. \tag{3.22}$$

Eliminating the time t from (3.19) and (3.21) gives us a relation between v and the acceleration length z:

$$z = (2/3)\,(m_0\, v_0^2/F)\,[(v/v_0)^{3/2} - 1]\,. \tag{3.23}$$

For $v \gg v_0$ this is asymptotically

$$z \rightarrow (2/3)\,(m_0\, v_0^{1/2}/F)\, v^{3/2}\,. \tag{3.24}$$

The time dependence of the rocket mass, $m = m(t)$ is given by (3.17) if we insert $v = v(t)$ given by (3.19). The result is

$$m/m_0 = [1 + (F/2\, m_0\, v_0)\, t]^{-1}\,, \tag{3.25}$$

with the asymptotic form

$$m/m_0 \rightarrow (2\, m_0\, v_0/F)\, t^{-1}\,. \tag{3.26}$$

From (3.25) we find for the rate the rocket loses mass:

$$\mathrm{d}m/\mathrm{d}t = -(F/2\, v_0)\,[1 + (F/2\, m_0\, v_0)\, t]^{-2} \tag{3.27}$$

with the asymptotic limit

$$\mathrm{d}m/\mathrm{d}t \rightarrow -(2\, m_0^2\, v_0/F)\, t^{-2}\,. \tag{3.28}$$

The power needed to drive the projectile is given by

$$P = F\, v = A(H^2/4\,\pi)\, v\,. \tag{3.29}$$

Finally, the propulsion efficiency is given by

$$\eta = \frac{(1/2)\, m\, v^2}{(1/2)\, m_0\, v_0^2 + \int_0^t P\, \mathrm{d}t}\,. \tag{3.30}$$

With $P\,\mathrm{d}t = Fv\,\mathrm{d}t = -2v^2\,\mathrm{d}m$, (3.30) can be brought into the form

$$\eta = \left[(m_0\, v_0^2/m\, v^2) + (4/m v^2) \int_m^{m_0} v^2\, \mathrm{d}m\right]^{-1}. \tag{3.31}$$

With the help of (3.17) we then find

$$\eta = 3/[4 - (m/m_0)^3]\,. \tag{3.32}$$

In the limit $m/m_0 \rightarrow 0$ one has $\eta \rightarrow 3/4 = 75\%$. (For smaller mass ratios m_0/m, η is larger.) The electromagnetic rocket gun has a lower efficiency than a pure electromagnetic gun, but the price in a lower efficiency is well paid because it prevents the vaporization of the projectile.

4. Thermodynamic Considerations

The travelling magnetic wave induces surface currents in the metallic encasement of the propellant and which are dissipated into heat. The heat is thermally conducted into the solid hydrogen and which evaporates. The hydrogen is thereby heated at the rate

$$\varepsilon = (j^2/\sigma)\,(2\,\pi\, r\,\delta/\pi\, r^2) = (j^2/\sigma)\; 2\,\delta/r\,, \tag{4.1}$$

where δ is the thickness of the metallic encasement, r its radius and σ its electrical conductivity. According to Maxwell's equations

$$j \simeq c\, H/4\,\pi\,\delta \tag{4.2}$$

by order of magnitude, and the heating of the hydrogen is given by

$$\varepsilon \simeq (H^2/8\,\pi)\,(c^2/\pi\,\sigma\, r\,\delta) = \varrho_{\mathrm{H}}\, c_{\mathrm{p}}\, \partial T/\partial t\,, \tag{4.3}$$

where $\varrho_{\mathrm{H}} = 0.07\ \mathrm{g/cm^3}$ is the density of solid hydrogen and c_{p} its specific heat at constant pressure. If v_{ev} is the evaporation velocity, we can put

$$\partial T/\partial t \simeq v_{\mathrm{ev}}\, (\nabla T/T)\, T_0 \simeq v_{\mathrm{ev}}\, T_0/r\,, \tag{4.4}$$

where T_0 is the evaporation temperature. Continuity requires that

$$\varrho_{\mathrm{H}}\, v_{\mathrm{ev}} = \varrho\, v\,. \tag{4.5}$$

With (4.4) and (4.5) we obtain from (4.3) a condition for the conductivity, resp. thickness δ of the encasement:

$$\sigma\,\delta = (H^2/8\,\pi)\,(c^2/\pi\,\varrho\, v\, c_{\mathrm{p}}\, T_0)\,. \tag{4.6}$$

Equation (4.6) ensures that the vaporization rate matches the mass flow rate of the jet, with the jet having the same speed as the projectile. Therefore, if (4.6) is satisfied, then also the assumption following (3.14), that the jet velocity is equal to the projectile velocity, is satisfied.

For the example, $H = 10^5$ G, $\varrho = 4 \times 10^{-6}\ \mathrm{g/cm^3}$, $v = 10^7$ cm/sec and $c_{\mathrm{p}}\, T_0 \sim 10^{10}$ erg/g, we find $\sigma\,\delta \sim 3 \times 10^{17}$ cm/sec. Therefore, if $\delta \simeq 0.1$ cm, one must have $\sigma \simeq 3 \times 10^{18}\ \mathrm{sec^{-1}}$. This large conductivity can be reached with ordinary conductors at cryogenic temperatures.

The evaporation velocity can be computed from (4.5). For the example $\varrho = 4 \times 10^{-6}\ \mathrm{g/cm^3}$, $v = 10^7$ cm/sec and $\varrho_{\mathrm{H}} = 7 \times 10^{-2}\ \mathrm{g/cm^3}$, we find $v_{\mathrm{ev}} \simeq 6 \times 10^3$ cm/sec.

The density and temperature of the plasma jet are determined by (3.7). One finds

$$\varrho = H^2/8\,\pi\, v^2 \tag{4.7}$$

and

$$T = v^2/3\, R\,, \tag{4.8}$$

where $R = 8.31 \times 10^7$ erg/g K is the universal gas constant. For the example $H = 10^5$ G, $v = 10^7$ cm/sec we find $\varrho \simeq 4 \times 10^{-6}$ g/cm^3, or $n \simeq 2.5 \times 10^{18}$ cm^{-3}, and $T \simeq 4 \times 10^5$ K. For the impact fusion velocity $v \simeq 200$ km/sec, we find $T \simeq 1.6 \times 10^6$ K and $\varrho \simeq 10^{-6}$ g/cm^3.

The part of the metallic encasement which contains the hydrogen is after its vaporization not more cooled and therefore will also be ablated. To compute the ablation velocity of the encasement, a similar calculation as for the evaporation velocity of the propellant can be made. The rate of the thermal energy released per unit volume in the encasement is

$$\varepsilon = j^2/\sigma \simeq c^2 H^2/(4\pi)^2 \sigma \delta . \tag{4.9}$$

The ablation velocity v_{ab} is related to the melting temperature T_{max} by

$$\varepsilon = \varrho_0 c_p \partial T/\partial t \simeq \varrho_0 c_p T_{max} v_{ab}/\delta , \tag{4.10}$$

where ϱ_0 is the density of the encasement and c_p its specific heat. From (4.9) and (4.10) one then can compute v_{ab}:

$$v_{ab} = (H^2/8\pi)(c^2/2\pi\sigma\delta\varrho_0 c_p T_{max}) . \tag{4.11}$$

At the high melting temperature the conductivity is smaller, and we assume that it is $\sigma \sim 10^{16}$ sec^{-1}. Using this value of σ we find for the example $H = 10^5$ G, $\varrho_0 c_p T_{max} \sim 10^{10}$ erg/cm^3 and $\delta \simeq 0.1$ cm, that $v_{ab} \simeq 10^4$ cm/sec. By order of magnitude this is about the same as v_{ev} and which ensures that the encasement is rapidly ablated after the hydrogen propellant has been vaporized.

5. Tensile Strength Considerations

The general limitation set by the tensile strength is expressed by (1.10). It does not take into account that the projectile has a propellant surrounded by an encasement. Even though the liquid or solid propellant can withstand large compressive forces, stresses resulting from shear must be balanced by the encasement. We will therefore supplement (1.10) by a more detailed estimate for the required thickness of the encasement.

The thickness δ, of the encasement must be large enough to withstand the magnetic pressure. If σ_s is the tensile strength of the encasement, the maximum pressure force per unit length it can withstand is

$$F_s = 2\delta \cdot \sigma_s . \tag{5.1}$$

The magnetic pressure force acting on the outer part of the encasement excerts a force per unit length given by

$$F_m = \frac{H^2}{8\pi} 2r . \tag{5.2}$$

The encasement will hold against the magnetic pressure as long as $F_s > F_m$. From this condition one obtains

$$\delta/r > H^2/8\pi\sigma_s . \tag{5.3}$$

For our example, $H = 10^5$ G, and putting $\sigma_s \simeq 10^{10}$ dyn/cm^2 (valid for steel), we find $\delta/r > 4 \times 10^{-2}$. Therefore, if $r \sim 1$ cm, then $\delta > 0.4$ mm, and which compares well with our assumed value $\delta \sim 1$ mm.

6. An Example

As a typical example for impact fusion we choose $v = 200$ km/sec, $v_0 = 1$ km/sec and $m = 1$ g. At this velocity the kinetic energy of a 1 g projectile is 20 MJ. It then follows from (3.17) that $m_0 \simeq 14$ g. For the initial length of the projectile we may assume $l \sim 10$ cm. With $\sigma_s \simeq 10^{10}$ dyn/cm^2 and $\varrho \simeq 1$ g/cm^3, we find from (1.10) that $a = a_{max} = 10^9$ dyn/cm^2, with $H_{max} \simeq 350\,000$ G. For a cross section $A = 3$ cm^2, we find $F \simeq 10^{10}$ dyn. The length of the accelerator, given by (3.24), is $z \simeq 260$ meter.

At $v = 2 \times 10^7$ cm/sec one computes from (4.7) $\varrho \simeq 10^{-5}$ g/cm^3, or $n \simeq 10^{19}$ cm^{-3}, and from (4.8) $T \simeq 10^6$ K.

Appendix I

If a plasma of finite conductivity is shock-heated and mixed with a magnetic field, eddy currents dissipated within the plasma heat up the plasma until the magnetic field energy becomes equal to the internal plasma energy. This fact, expressed by (3.7) can be easily proved.

The mixing and resistive heating can occur in a number of ways. The most important modes of dissipation are turbulence and shock waves, the first one for subsonic and the second one for supersonic flow of the propellant relative to the magnetic field. Furthermore, in the presence of magnetic fields, collisionless shock waves are possible which imply an anomalous high resistivity of non-collisional origin.

Using electrostatic cgs units, the resistive dissipation rate per unit volume is given by

$$\varepsilon = j^2/\sigma\,, \qquad \text{(A.I.1)}$$

where j and σ are the plasma current density and conductivity. The diffusion of the magnetic field into the plasma is given by

$$\frac{\partial \boldsymbol{H}}{\partial t} = \frac{c^2}{4\pi\sigma}\nabla^2 \boldsymbol{H} \qquad \text{(A.I.2)}$$

which leads to the characteristic diffusion time

$$\tau = 4\pi\sigma\lambda^2/c^2\,, \qquad \text{(A.I. 3)}$$

where λ is a characteristic length over which the magnetic field changes. In case of anomalous resistive dissipation λ is a measure for the shock thickness.

The total energy which can be dissipated is

$$e = \varepsilon\,\tau = 4\pi\,(j\,\lambda/c)^2\,. \qquad \text{(A.I.4)}$$

With the help of Maxwell's equation (3.2),we find that

$$e \sim H^2/8\pi\,. \qquad \text{(A.I.5)}$$

A rigorous albeit less transparent treatment of the same problem gives $e = H^2/8\pi$. For a singly ionized hydrogen plasma $e = 3nkT$ and we thus obtain (3.7).

Appendix II

Here we compute the maximum velocity attainable for a projectile in contact with the wall, as it is the case for a railgun.

We consider a projectile of rectangular cross section D^2, separated by a clearance δ from the wall of the accelerating structure. To be in physical contact with the wall, in reality means that an arc is burning over the distance δ in between the projectile and the wall. This is the situation which always exists for a sliding contact between two conductors.

The radiative energy flux emitted from the arc having the temperature T_a is given by

$$\Phi_r = 4D\,\delta\varkappa T_a^4\,, \qquad \text{(A.II.1)}$$

where $\varkappa = 5.75 \times 10^{-5}$ erg/cm^2 sec K^4 is the Stefan-Boltzmann constant. At very high projectile velocities, above 10 km/sec, a lower estimate for the temperature of the arc plasma, neglecting heat conduction, viscous friction and joule heating, is given by the adiabatic wall temperature T_a. It is the temperature the plasma in between the walls of the sliding contacts will assume by coming to rest on the surfaces of these rapidly moving walls. It is given by

$$T_a = v^2/2c_p\,, \qquad \text{(A.II.2)}$$

where

$$c_p = (5/2)\,(Z+1)\,R/A \qquad \text{(A.II.3)}$$

is the specific heat at constant pressure of a Z-times ionized plasma of atomic weight A. $R = 8.31 \times 10^7$ erg/K is the universal gas constant. Inserting numerical values we have

$$T_a = 2.4 \times 10^{-9} A\, v^2/(Z+1)\,. \qquad \text{(A.II. 4)}$$

Substituting (A.II.4) into (A.II.1) we obtain the radiative energy loss

$$\Phi_r = 7.6 \times 10^{-39} D\,\delta v^8\,(A/(Z+1))^4\,. \qquad \text{(A.II.5)}$$

Equating this radiative energy loss with the power to drive the projectile

$$\Phi_p = (H^2/8\pi)\,D^2 v \leqq \sigma_s D^2 v\,, \qquad \text{(A.II.6)}$$

we obtain a condition for the maximum value of the velocity:

$$v_{max} = 2.8 \times 10^5\,(\sigma_s D/\delta)^{1/7}\,((Z+1)/A)^{4/7}\,. \qquad \text{(A.II.7)}$$

Putting $\sigma_s = 10^{10}$ dyn/cm^2, $D/\delta \simeq 10$ and $(Z+1)/A \simeq 0.1$, we find $v_{max} \simeq 30$ km/sec.

The dependence of v_{max} on Z and A suggests that it would be ideal to put a hydrogen plasma in between the sliding contacts. There $(Z+1)/A = 2$, and $v_{max} \simeq 150$ km/sec. However, at the high contemplated velocities, it is unlikely that the arc plasma would remain a pure hydrogen plasma, mainly because material from the sliding wall would be ablated at a substantial rate and would mix with the hydrogen plasma.

Appendix III

To compute the maximum velocity attainable by the magnetic acceleration of an ordinary conductor, we start with the magnetic body force density in the conductor given by

$$\boldsymbol{f} = (1/c)\,\boldsymbol{j} \times \boldsymbol{H}\,, \qquad \text{(A.III.1)}$$

where $\boldsymbol{j}$ is the current density vector. With Maxwell's equation (3.2), one obtains from (A.III.1)

$$f = (4\pi\, l/c^2)\, j^2\,, \tag{A.III.2}$$

where l is a length which, up to a factor of order unity, is equal to the linear dimension of the conductor. The rate of the resistive energy dissipation in the conductor is

$$\varepsilon = j^2/\sigma\,, \tag{A.III.3}$$

where σ is the conductivity. Due to this resistive heating, the conductor melts after the time t_0 given by

$$\varrho\, c_p\, T_{max} = \varepsilon\, t_0\,, \tag{A.III.4}$$

where ϱ is its density, c_p its specific heat at constant pressure and T_{max} its melting temperature. The acceleration of the conductor is given by

$$a = f/\varrho = 4\pi\, l\, j^2/\varrho\, c^2 \tag{A.III.5}$$

and therefore the maximum velocity

$$v_{max} = a\, t_0 = (4\pi\, \sigma\, l\, c_p\, T_{max})/c^2\,. \tag{A.III.6}$$

At temperatures near the melting point the conductivity is comparatively low, even if it was high at normal temperatures. We may therefore assume that $\sigma \sim 10^{16}\,\text{sec}^{-1}$. Then, for the example $c_p\, T_{max} \sim 2\times 10^9$ erg/g, and $l \sim 1$ cm, we find $v_{max} \sim 2$ km/sec.

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